LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PLANT BIOLOGY & PLANT BIOTECH.**

FOURTH SEMESTER – **JUNE 2015**

ST 4209 - MATHEMATICAL STATISTICS

Date : 03/07/2015 Dept. No. Max. : 100 Marks
Time : 10:00-01:00

Section A

Answer all the questions **10 x 2 = 20 marks**

1. Provide the axiomatic definition of probability.
2. If P(A) =0.7 , P(B) =0.5 and P $(A \cup B)$ = 0.8 find $P(A \cap B^c)$.
3. Find E(X) if f(x) = 6x (1-x) , $0 \leq x \leq 1$, zero elsewhere.
4. If 12 fair coins are flipped simultaneously find the probability of getting at least 4 heads.
5. Write the moment generating function and variance of binomial distribution.
6. Define beta distribution of second kind.
7. Let X have the probability density function $f(x) = x^2/9$, 0<x<3 , zero elsewhere find the the probability density function of $Y = X^3$.
8. Define chi-square distribution with n degrees of freedom.
9. State the sufficient conditions for an estimator to be consistent.
10. Define Type I and Type II errors in testing of hypothesis.

Section B

Answer any five questions **5x 8 =40 Marks**

11. (a) State and prove Bayes' theorem .
    (b) State and prove addition theorem on probability for three events. **(4+4)**
12. If P(A) = 1/3 , P(A) = 1/5 and $P(A \cap B)$ = 1/ 7 find (i) $P(A|B)$ (ii) $P(A^c|B)$ (iii) $P(A|B^c)$ (iv) $P(A^c|B^c)$
13. If $f(x) = (1/2)^x$, x = 1,2,3, … , zero elsewhere compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$.
14. Derive the moment generating function of Poisson distribution. Also find mean and variance.
15. If f(x,y) = 8xy , 0<x<y<1 , zero elsewhere find (i) $P(X < 1/2 \cap Y < 1/4)$
    (ii) the marginal and conditional distributions.
16. Derive the mean and variance of beta distribution of II kind.
17. State and prove Boole's inequality.
18. If $X_1, X_2, \ldots, X_n$ is a random sample from $N(\theta, 1)$, $\theta \epsilon (-\infty, \infty)$ find the maximum likelihood estimator of θ.

Section C

Answer any two questions **2x 20 = 40 Marks**

19. (a) State and prove Chebyshev's inequality.

(b) A random variable X has the following probability distribution:

x	:	0	1	2	3	4	5	6	7	8
p(x)	:	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k. (ii) Find $P(X<3)$ and $P(0<X<5)$

(iii) What is the smallest value for which $P(X \leq x) > 0.5$?

(iv) Find the distribution function of X. **(10+10)**

20. (a) Show that under certain conditions binomial tends to Poisson distribution.

(b) Establish the additive property of gamma distribution.

(c) If X is $N(30, 5^2)$ find (i) $P(26<X<40)$ (ii) $P(|X-30|>5)$ (iii) $P(X>42)$ (iv) $P(X<28)$

(4+4+12)

21. If $f(x_1,x_2) = 21x_1^2x_2^3$, $0 < x_1 < x_2 < 1$, zero elsewhere, find the conditional mean and variance of X_1 given $X_2=x_2$, $0<x_2<1$ and X_2 given $X_1 = x_1$, $0<x_1<1$.

22. (a) Derive the probability density function of F distribution.

(b) Derive the mean and variance of chi-square distribution with n degrees of freedom.

(10 +10)

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